AFOSR 4706

63-4-2

Grant No. AF-AFOSR 62-200

FINAL REPORT FOR 1962

INVESTIGATION OF PLATES AND SHELLS UNDER EXTERNAL LOADING AND ELEVATED TEMPERATURES

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April 1963

PIBAL Report No. 664

This research was supported by the Air Force Office of Scientific Research of the Air Research and Development Command under Grant No. AF-AFOSR 62-200.

This research was supported by the Mechanics Division, AFOSR, SREM.
under ~~Contract~~/Grant AFOSR 62-200
SREM-1

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CONTENTS

*Numbers in brackets designate references.

INTRODUCTION

The studies briefly described in this report stem from continuing investigations of plates and shells under external loading and elevated temperatures, and include problems of special interest to designers of missiles and aircraft. This work was performed during the past year in the Department of Aerospace Engineering and Applied Mechanics of the Polytechnic Institute of Brooklyn on AFOSR Grant 62-200. A summary of the previous four years of work in this area has been given in PIBAL Rep. No. 599 (AFOSR 2280).

As indicated by the table of contents, the research covered several areas and a variety of problems. Chapter I presents the results of investigations of the effects of creep in structures, with particular emphasis on the bending of circular plates. Chapter II outlines the work performed on heat conduction problems using Biot's variational method. Chapter III discusses the problem of the buckling (small-deflection theory) and post-buckling (large-deflection theory) of noncircular (oval) cylindrical shells under axial compression. Chapter IV describes work on the analysis of the effects of concentrated loads applied to reinforcing frames of finite and infinitely long circular cylindrical shells. Chapter V discusses the results obtained from the analysis of the dynamic response of plastic spherical shells

The list of references at the end of this report represents reports, publications, and theses prepared during the course of the grant period. The writer would like to take this opportunity to thank those authors whose names appear on this list for their invaluable contributions to the work performed on this grant. Also, he would like to acknowledge the financial assistance and encouragement provided by the Air Force Office of Scientific Research throughout the course of the studies summarized herein.

I. CREEP OF STRUCTURES [2,3,6,7]

In the work during the past year on the inelastic behavior of structures a creep law governing the behavior of compressible materials was formulated on the basis of linear stress-strain relations of classical elasticity and the analogy between creep and nonlinear elasticity. The strain and complementary energy functions associated with the analogous elastic law were developed. Also, the nonlinear elastic law was used in deriving the moment curvature relations governing the lateral bending of thin plates. These relations were applied to creep bending analyses of compressible circular plates. The method of solution was based on an iterative procedure applicable in general to such plates subjected to radially symmetric lateral loads. Numerical solutions for moments and deflections were obtained which are applicable to simply supported and clamped edge circular plates under uniformly distributed loads for a wide range of the parameters involved.

The creep analyses described above have been continued. The iterative procedure developed and successfully applied to full circular plates with lateral loads is currently being extended for annular plates. This procedure is considerably more involved than that applied in the above analysis. The reason for the additional complexity is that not only the edge moment and the exponent n of the nonlinear creep law enter as variables, but also the edge shear as well as the deflections.

II. HEAT CONDUCTION [5]

Previously reported investigations on heat-conduction problems (PIBAL Rep. Nos. 587, 595) were extended and unified, and a paper [5] was prepared in which Biot's variational principle is applied to a number of different one-dimensional heat conduction problems. These examples show the applicability of the variational principle to problems involving prescribed heat flux boundary conditions as well as to problems with temperature-dependent material properties. In this work the equivalence of the result obtained in applying the variational principle for a prescribed surface temperature history to that obtained for a prescribed heat flux is revealed.

These studies are currently being extended to the problem of the determination of the temperature distribution in the region exterior to a spherical cavity whose surface temperature is constant. Both Biot's variational principle and Goodman's heat-balance integral technique are being applied.

III. BUCKLING AND POSTBUCKLING OF NONCIRCULAR CYLINDRICAL SHELLS [4,9]

Investigation of the buckling and postbuckling (nonlinear) characteristics of noncircular (oval) cylindrical shells continued. The work on the analysis and numerical computations for the problem of the classical buckling (small-deflections) of long oval cylindrical shells under axial compression was completed. Buckling stresses were obtained for a range of major-minor axis ratio b/a of the oval cross section of 1 to 2 as well as a range of the ratio of average radius to thickness r_o/t of 10 to 1500.

The results show that for a given value of the latter ratio the buckling stress decreases rapidly with increasing eccentricity of the cross section. Thus, according to small-deflection theory, the oval cylinder can be significantly weaker than an "equivalent" circular cylinder having the same wall thickness and circumferential length. For example, when $r_o/t \simeq 300$ the ratio of the buckling stress of the oval cylinder to that of the equivalent circular cylinder is 0.52 when b/a = 1.4 and 0.07 when b/a = 2. Furthermore, determination of the eigenvectors corresponding to the buckling stresses reveals that the distortion of the oval cylinder is localized about the ends of the minor axis, i.e., in the neighborhood of the maximum radius of curvature of the cross section. Also, for a given value of the major-minor axis ratio b/a the deformation becomes more localized as the average radius-thickness ratio r_o/t increases.

The results also show that within the framework of classical theory a lower bound solution which corresponds to the introduction of the maximum radius of curvature in the formula for the classical buckling stress of a circular cylindrical shell yields good results for moderate eccentricities, particularly for very thin shells.

In the associated investigation of the postbuckling characteristics of noncircular cylindrical shells, nonlinear (large-deflection) theory is being applied. Just as in the problem of the buckling of circular cylinders, such analyses are necessary for a thorough understanding of the buckling characteristics as well as to indicate the applicability of and limitations on the classical buckling theory. In the present work a finite trigonometric series for the deflection function is being used within the framework of a Rayleigh-Ritz procedure. The selection of the series has been based upon that which was required for sufficient accuracy in the above described classical buckling analysis as well as upon the function applied successfully to the large-deflection analysis of the circular cylinder (see Kempner, J. Aero. Sci., Vol. 21, No. 5). In addition terms are introduced which Almroth (AIAA Journ., Vol. 1, No. 3) has shown recently to afford a significant increase in the accuracy of the circular cylinder problem. The resulting expression for the deflection function contains many more degrees of freedom than that required for the circular cylinder solution, since it is no longer possible to assume that a single characteristic wave length governs the deflected shape in the circumferential

direction. Rather, the wave shape in this direction is represented by a finite Fourier series. Such a representation is required because of the relatively localized nature of the buckles. It has, however, the advantage that continuity of deformation is ensured. This condition is only met in an approximate manner by available solutions of the circular cylinder.

In this work the energy procedure has resulted in a set of simultaneous nonlinear polynomial equations each of the third degree in the undetermined deflection amplitudes. The total number of equations requiring inclusion in this set is dependent upon both the major-minor axis ratio and the average radius-to-thickness ratio. When these equations are linearized, the corresponding small-deflection buckling solution referred to previously results.

Investigation of several possible methods of solving the nonlinear equations led to the conclusion that the method of steepest descent holds the greatest promise of success. Accordingly, an IBM-7090 machine program was developed to solve the cubic matrix equations for given values of the geometric parameters as well as for a given value of the axial wave length of buckles. The latter quantity must be varied until the solution of the matrix corresponds to the minimum of the total potential with respect to the deflection amplitudes and the axial wave length.

As a first step in the investigation of the convergence of the computing program, a circular cylinder was considered ($b/a = 1$). For a single r_o/t ratio, a complete set of calculations was performed successfully to yield the postbuckling axially compressive stress as a function of the unit end shortening. The results yielded a minimum postbuckling load equal to approximately 15% of the classical buckling load. This result is in good agreement with one of the recent results given by Almroth for the circular cylinder.

In addition, preliminary numerical results from the nonlinear analysis of the oval cylinders reveal postbuckling behavior analogous to that of the circular cylinder. However, the ratio of the minimum postbuckling load to the classical buckling load for the oval cylinder is greater than the corresponding ratio for the circular cylinder. For example, from completed calculations for an oval cylinder corresponding to $r_o/t = 11$ and $b/a \simeq 1.5$, the former ratio is 0.35, while the latter is 0.15. Work is currently being directed towards the large-deflection analysis for a range of r_o/t up to 1500, while b/a varies between 1 and 2.

IV. STRESS AND DISPLACEMENT ANALYSES OF REINFORCED CIRCULAR CYLINDRICAL SHELLS [1]

This chapter describes the work performed during the past year on the elastic analysis of circular cylindrical shells, reinforced at their midlength by a frame subjected to concentrated loads and moments.

Reinforced shells are used so extensively in the fields of submarines, missiles, aircraft, pressure vessels, to mention the most prominent, that new investigations in this area are of continuing interest.

Initially, the problem of an infinitely long circular cylindrical shell reinforced at its midlength by a single ring to which is applied two self-equilibrating concentrated radial forces and moments in the plane of the ring has been solved.

Since the analytical development of this problem set a pattern for the subsequent work in this area, a brief outline of this analysis of the ring-shell problem follows.

The shell was assumed to be loaded by unknown line loads, the reactions caused by the ring, and the solution of the shell equations was determined as a function of the unknown line loads (expressed as Fourier series with unknown coefficients).

Because the Donnell shell equations were expected to lead to poor results for the lower harmonics, the analysis of the shell was also carried out using the Flügge shell equations and the Love-Reissner shell equations. The characteristic equation resulting from the Flügge or Love-Reissner differential equations is considerably more cumbersome than that from the Donnell equations and for harmonics $n > 1$ has not been solved in closed form.

In treating the uniform ring, the loading was taken to consist of the radial and circumferential reactions from the shell (again expressed as a Fourier series with the same unknown coefficients) plus the two self-equilibrating diametrically-opposed concentrated radial loads and concentrated moments in the plane of the ring. The ring was considered on the basis of deep ring theory. The solution yielded the ring displacements and stresses in terms of the interaction load coefficients.

To complete the solution of the ring-shell problem the ring and shell solutions were matched leading to an infinite number of sets of simultaneous equations. Each of these sets contained two equations in two unknowns. From these the coefficients of the trigonometric interaction load series for the corresponding harmonics were obtained.

Since the stresses and displacements in the ring and shell were expressed as explicit functions of the interaction load coefficients, the complete interaction problem could be considered solved.

The stresses and displacements were obtained for a single set of parameters. It was found that in the neighborhood of the ring the axial bending stresses due to the concentrated loads were high. These stresses were found to be highest under the load, lowest approximately 45° from the load and reversed in sign 90° from the load. For the parameters chosen, it was found that the length of shell which supports the load was approximately one third the radius.

The circumferential bending stress due to the concentrated loads was also highest under the load, almost zero 45° from the load, and reversed in sign 90° from the load. The maximum circumferential bending stress was found to be almost one third the maximum axial bending stress.

The conclusions reached for the axial and circumferential bending stress in the shell due to the concentrated radial loads also apply in the main to the reinforced shell subjected to concentrated self-equilibrating moments.

The maximum circumferential membrane stresses due to both the concentrated loads and moments are highest under the loads, very small 45° from the loads, and reversed in sign 90° from the loads. These were found to be of the order of magnitude of the bending stresses.

The magnitude of the membrane shear stresses due to the concentrated radial forces were found to be less than the normal stresses. Under the load and 90° from the load these stresses, because of symmetry, were identically zero. The shear stress due to the concentrated moments becomes infinite under the load. This occurs because there is a discontinuity in the circumferential stress at this point. An infinite shear stress under the load is required to maintain equilibrium of an element including the external moment.

The agreement between the results obtained using the Donnell shell equations, the Love-Reissner shell equations, and the Flügge shell equations were excellent for all harmonics.

A second problem, similar to the one discussed above, was initiated. Two finite circular cylindrical shells, one with simply supported ends and the other with fixed ends, reinforced at their midlength by a single uniform ring subjected to a radial force, a tangential force, and a concentrated moment all in the plane of the ring were analyzed.

The method of analysis here follows the procedure outlined in the discussion of the previous problem, i.e., the infinite shell. Numerical work is now being undertaken with respect to this problem. An IBM 650 computer is being used to obtain results.

V. DYNAMICS OF PLASTIC SPHERICAL SHELLS [8]

Work also continued on the studies of the response of shell structures to suddenly applied loads which exceed the load corresponding to the yield strength of the material. In this work the material of the shell is assumed to obey Tresca's yield condition and the associated flow rule.

Some of the results obtained up to the present time appear in [8] where the behavior of complete spherical shells and simply supported spherical caps under impulsive loading by a uniform external pressure is discussed. The pressures are assumed to be greater than the static collapse pressures and to act for a short period of time. The displacements of the shells are presented for particular values of the parameters of the problem.

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